Name: _____ **Teacher: MA LD SW LR RC GS**

NESA Number: _____

ASCHAM SCHOOL

MATHEMATICS ADVANCED TRIAL EXAMINATION 2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black non-erasable pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper

Section I **10 marks**

- Answer Questions 1–10 using the Multiple Choice sheet
- Allow about 15 minutes for this section.

Section II **90 marks**

- Attempt Questions 11–32.
- Allow about 2 hours 45 minutes for this section.
- For questions in Section II, show relevant mathematical reasoning and/ or calculations.
- Additional writing space is available on pages 27 to 32. If you use this space, please start a new sheet of paper for each question.

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

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1. The first 3 terms of an arithmetic sequence are $-3, 1, 5$. What is the 12^{th} term?

(A) 30 (B) 37 (C) 38 (D) 41

2. The table shows the width of a pond at regular intervals along its length.

Distance	0	10	20	30	40	50
Width (m)	1.2	1.35	1.55	1.9	1.65	1.4

Use the trapezoidal rule to find the approximate area of the pond rounded to the nearest square metre.

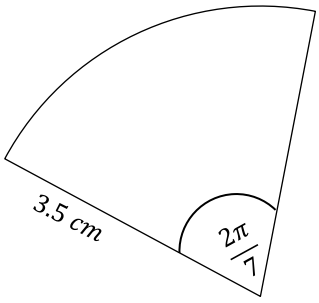
(A) 61 m^2 (B) 65 m^2 (C) 78 m^2 (D) 91 m^2

3. Which of the quadratic equations has 2 distinct real and rational roots?

(A) $y = -x^2 - 3x + 5$ (B) $y = 4x^2 - x - 5$

(C) $y = -x^2 - 4$ (D) $y = 2x^2$

4. What is the exact perimeter of the sector shown below?



(A) $\pi\ cm$ (B) $\frac{49}{4}\pi\ cm$ (C) $\left(\frac{2}{7}\pi + 7\right) cm$ (D) $(\pi + 7)\ cm$

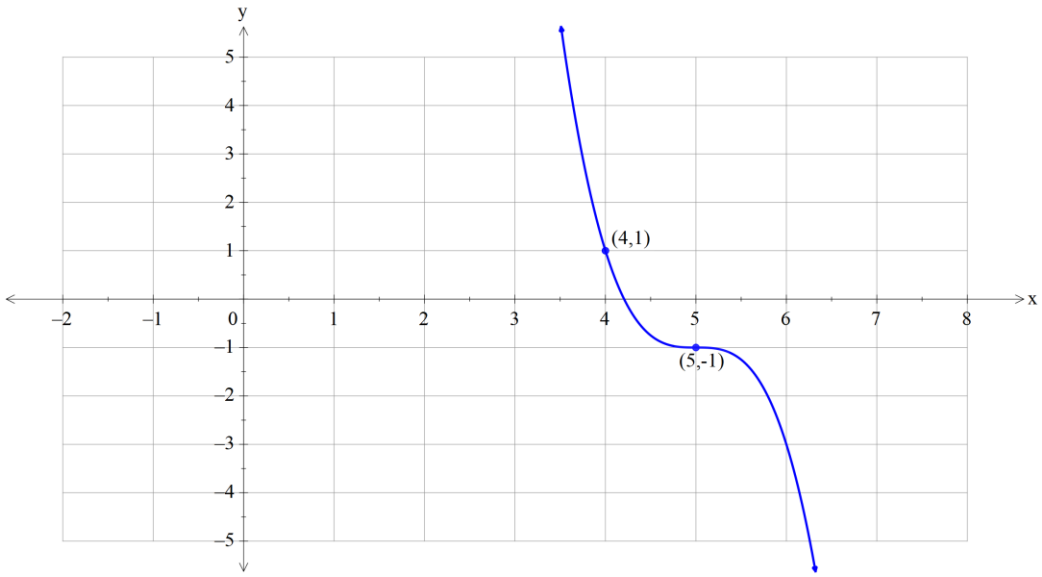
5. The point $P(-2,3)$ lies on $y = f(x)$. What are the coordinates of the image point, P' on the graph of $y = 3f(x-4)+1$

(A) $(2,10)$ (B) $(-6,10)$ (C) $(-6,2)$ (D) $(2,0)$

6. Pearl invests \$12000 into a saving account that offers an interest rate of 4.15% p.a. compounded yearly. Gary invests \$12000 into an account with a different interest rate that compounds monthly. What interest rate as a $\%\ p.a.$ does Gary's bank offer if after 4 years both Pearl and Gary have the same amount in their accounts?

(A) $4.42\%\ p.a.$ (B) $4.07\%\ p.a.$ (C) $3.98\% p.a.$ (D) $3.04\%\ p.a.$

7. The cubic function graphed below has an equation in the form $f(x) = a(x-b)^3 + c$.



Which of the following statements is true about $f(x)$.

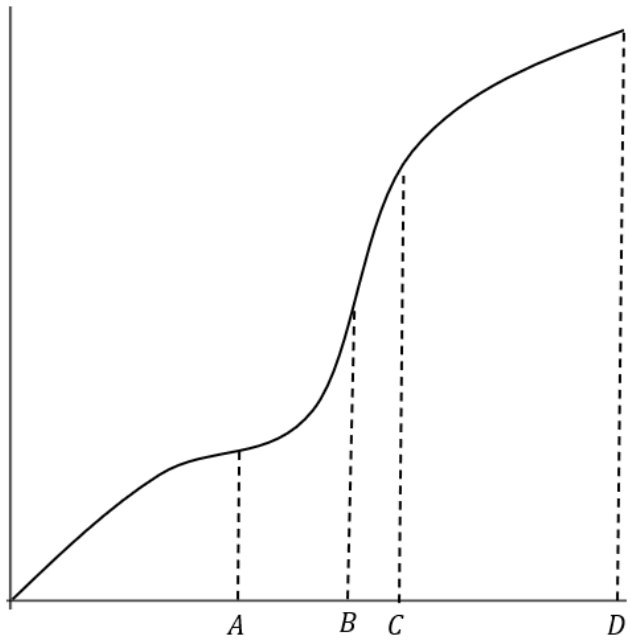
(A) $a < b < c$ (B) $a < c < b$

(C) $c < a < b$ (D) $b < c < a$

8. Find the value of $\int_0^5 |x-3|\,dx$.

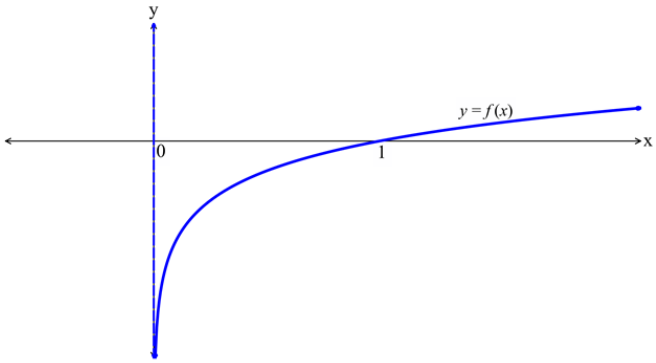
(A) -2.5 (B) 0 (C) 6.5 (D) 13

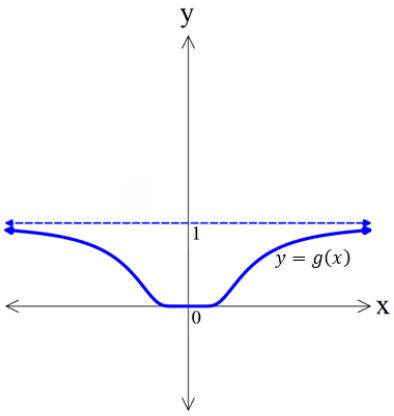
9. The cumulative distribution function for the continuous random variable X is graphed below. What is the mode of the probability density function?



(A) A (B) B (C) C (D) D

10. The graphs of $f(x)$ and $g(x)$ are shown below.





What is the range of $f(g(x))$?

(A) Range: $(-\infty, 0)$

(B) Range: $(0, \infty)$

(C) Range: $(-\infty, 1)$

(D) Range: $(1, \infty)$

End of Multiple Choice.

Mathematics Advanced

Section II Answer booklet

90 marks
Attempt Questions 11 – 32.
Allow about 2 hours and 45 minutes for this section.

Instructions:

- Answer each question in the spaces provided. These spaces provide guidance for the expected length of response.
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Question 11 (2 marks)

Solve $\frac{3x}{x+2}=4$. **1**

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Question 12 (4 marks)

Evaluate the following expressions.

(a) $\log_2 4x+\log_2 8x-2\log_2 x$ **2**

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(b) $\frac{25^x\times 3^{2x+5}}{15^{2x+1}}$ **2**

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Question 13 (2 marks)

Evaluate $\int \frac{x}{(2-x^2)^3} \, dx$. **2**

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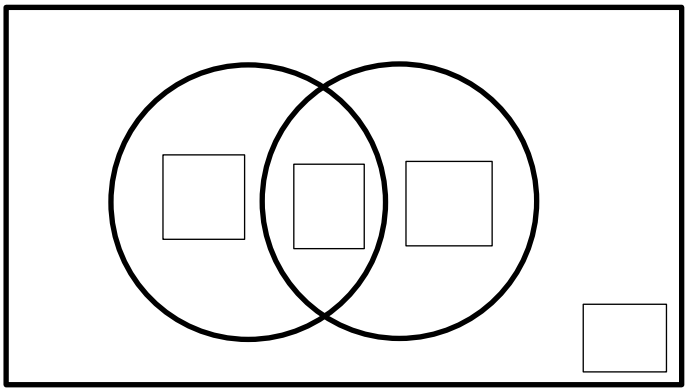
Question 14 (3 marks)

A survey of 150 people found that 113 people liked swimming, 55 liked surfing and 15 liked neither.

(a) Use this information to fill in the blank spaces in the Venn diagram. **2**



(b) If 2 people are selected at random, find the probability that both people like swimming only. **1**

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Question 15 (3 marks)

For what values of x is the function $f(x) = x^4 - 4x^3 - 48x^2$ concave down? **3**

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Question 16 (4 marks)

(a) Chloe sits a marine biology test and receives a mark of 60. The mean mark on the test was 50 with a standard deviation of 10. What percentage of her cohort did she score higher than? **2**

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(b) Chloe also sits a test on mythological creatures. The mean mark is 57 with a standard deviation of 13. Chloe received a mark of 71. Which test did she do better in compared with her cohort? Justify your answer. **2**

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Question 17 (6 marks)

A table of future value interest factors for an annuity of $1 is shown.

Table of future value interest factors

n/r	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%
4	4.06040	4.09090	4.12161	4.15252	4.18363	4.21494	4.24646	4.27819
8	8.28567	8.43284	8.58297	8.73612	8.89234	9.05169	9.21423	9.38001
12	12.68250	13.04121	13.41209	13.79555	14.19203	14.60196	15.02581	15.46403
16	17.25786	17.93237	18.63929	19.38022	20.15688	20.97103	21.82453	22.71934
20	22.01900	23.12367	24.29737	25.54466	26.87037	28.27968	29.77808	31.37142
24	26.97346	28.63352	30.42186	32.34904	34.42647	36.66653	39.08260	41.68920
28	32.12910	34.48148	37.05121	39.85980	42.93092	46.29063	49.96758	53.99333
32	37.49407	40.68829	44.22703	48.15028	52.50276	57.33450	62.70147	68.66625

(a) Ariel wants to have saved $10 000 by the end of 8 years. She will make regular instalments at the end of each 6-month period into a bank account with 4% interest p.a. compounded every 6 months. Find the value of each instalment that Ariel needs to make to reach her goal. **2**

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(b) Show that the amount in the account after 2 years is $2211.24. **1**

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(c) After 2 years, Ariel gets a promotion at work. She can now afford to make the same size instalments as in part (a) on a quarterly basis. The interest is still 4% p.a. but compounded quarterly. How much more than her goal of $10 000 will she have at the end of the 8 years? **3**

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Question 18 (3 marks)

Solve the equation $(2\cos x)^2 = 3$ in the domain $-\pi \leq x \leq \pi$. **3**

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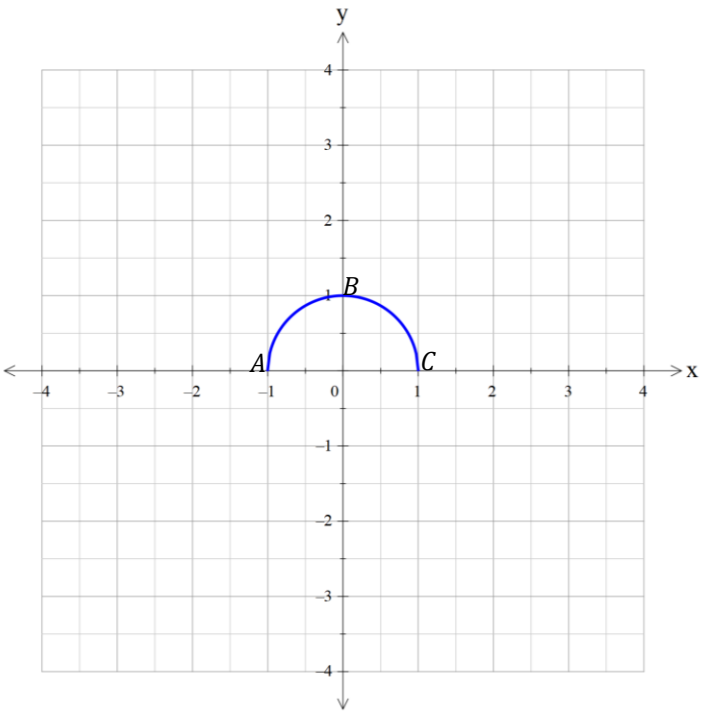
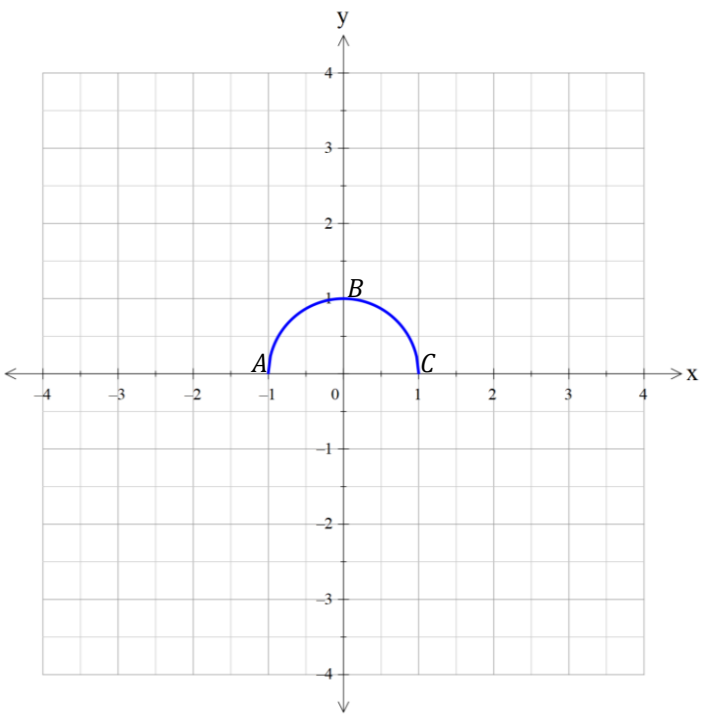
Question 19 (4 marks)

The function $y = f(x)$ is graphed below. On the same set of axes, sketch the transformed function clearly showing the transformed points of A, B and C.

(a) $y = 3f(x-1)$ **2**



(b) $y = f\left(\frac{1}{2}x\right) - 3$ **2**



Question 20 (5 marks)

(a) Find the equation of the normal to the curve $y = -x^2 + 4x - 1$ at the point $(1,2)$. **3**

(b) The normal intersects $y = -x^2 + 4x - 1$ again at point P. Find the coordinates of point P. **2**

Question 21 (3 marks)

Show that $\dfrac{1-\tan^2 x}{1+\tan^2 x} = 1-2\sin^2 x$. **3**

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Question 22 (2 marks)

A geometric sequence is formed by

$$f(x) + f'(x) + f''(x) + f'''(x) + \cdots$$

where $f(x) = e^{\frac{1}{2}x}$. Find the limiting sum in terms of x. **2**

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Question 23 (5 marks)

The population (P) of sea turtles over time (t) in years can be modeled using the equation

$$P = 30000e^{-kt} + 8000.$$

After 10 years, the population has decreased to 26196 turtles.

(a) What is the initial population of sea turtles? **1**

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(b) Show that the value of k is 0.05 rounded correct to two decimal places. **2**

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(c) Sketch the population of sea turtles over time. **2**

Question 24 (5 marks)

A ball is thrown upwards from the edge of a cliff and hits the ground below. Its vertical height (h) in metres above the ground is given by the formula $h = -t^2 + 2t + 3$ where t is the time in seconds.

(a) At what time does the ball hit the ground? **1**

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(b) At what time is the ball stationary? **1**

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(c) In the box below, graph the velocity function of the ball from the time it is launched until it hits the ground. **2**

(d) Find the total vertical distance travelled by the ball. **1**

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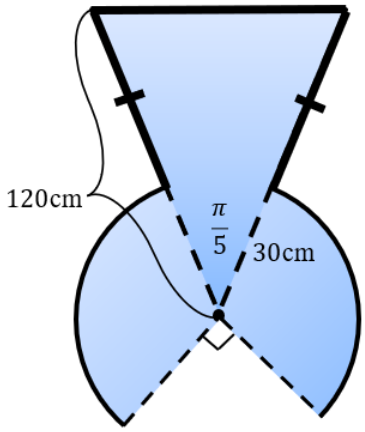
Question 25 (4 marks)

A mermaid tail pictured below is designed using the following:

- A large circle with a 30cm radius.
- An isosceles triangle with 2 sides of length 120 cm and an included angle of $\frac{\pi}{5}$.
- A sector with a 30 cm radius and an included angle of $\frac{\pi}{2}$ that has been removed from the larger circle.
- The vertex of the triangle and the sector meet at the centre of the circle.



NOT TO SCALE

Find the area of the mermaid tail (the shaded area), giving your answer correct to 1 decimal place. **4**

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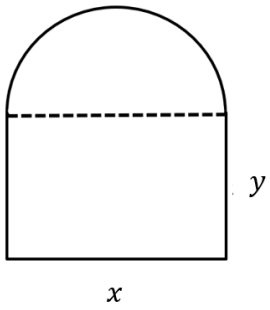
Question 26 (6 marks)
The cross section of a treasure chest is in the shape of a rectangle and a semicircle. The total perimeter of the cross section is 250 cm.



(a) Show that the area of the of the cross section is given by: **3**

$$A = 125x - x^2\left(\frac{\pi+4}{8}\right).$$

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(b) Find the maximum area of the cross-section of the treasure chest giving your answer correct to one decimal place. **3**

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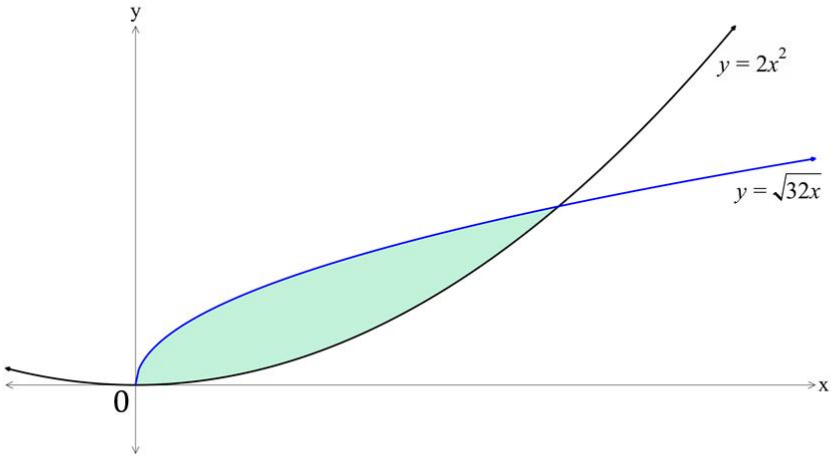
Question 27 (3 marks)

Find the area between the graphs of $y = 2x^2$ and $y = \sqrt{32x}$. **3**



Question 28 (5 marks)

Eugene deposits a sum of money into a bank account at the start of each year. The bank offers interest of 4% p.a. compounded every 6 months. Let A_n be the value of the account after n years and P be the value of each deposit.

(a) Show that the total value in the account after 3 years is given by: **2**

$$A_3 = P(1.02)^6 + P(1.02)^4 + P(1.02)^2 .$$

(b) Show that $A_n = \frac{2601}{101} P(1.02^{2n} - 1)$. **2**

Question 28 continues on the next page

(c) If Eugene saves \$500 000 dollars at the end of 25 years, find the size of his deposits. **1**

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Question 29 (4 marks)

(a) Differentiate $(x-1)(\sin x)$. **2**

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(b) Hence find $\int x\cos x \ dx$. **2**

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Question 30 (6 marks)

Seven scrabble tiles labelled with the letters A to G are placed in a bag. Each letter has a value assigned to it as shown in the photo below.

A player picks 2 tiles from the bag (1 at a time without replacement), and their score (X) is the sum of the numbers on the tiles.

(a) Complete the discrete probability distribution table by calculating the probability of getting each score. It may help to draw a sample space. **2**

Score	2	3	4	5	6	7
$p(X = x)$						

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(b) What is the expected score from playing this game? **2**

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Questions 30 continues on the next page

(c) Sebastian plays this game and scores a 5. What is the probability that he picked a tile with a vowel? **2**

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Question 31 (8 marks)

The depth (d) in metres of the water in a wave pool varies as waves are sent out. The depth is measured from the floor and can be modelled by the equation $d = \frac{4}{5}\sin at + \frac{3}{2}$, where t is the time in seconds and a is a constant. Each wave has a period of 2.5 seconds measured from the peak of the wave to the peak of the next wave.

(a) Find the exact value of a. **1**

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(b) Find the maximum height of the wave. **1**

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(c) Sketch a graph showing the depth of the water over the first 5 seconds. **3**

(d) Sandy can only touch the bottom of the pool when the height of the wave is less than 1.7m. For what percentage of the first 5 seconds can she touch the bottom of the pool? Give your answer correct to 1 decimal place. **3**

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Question 32 (4 marks)

The lengths of fish in metres in the harbour are distributed according to the equation,

$$f(x) = \begin{cases} -2x(x^2-1)^3 + 0.5 & for \quad 0 < x \leq 1 \\ -\dfrac{x-2}{2} & for\ 1 < x < 2 \\ 0 & for\ x > 2 \end{cases}$$

People are only permitted to catch a fish whose length is between 0.45 m and L m long. If 60% of fish in the harbour are within this legal limit, find the value of L correct to 2 decimal places. **4**

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End of Examination

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Label each question carefully.

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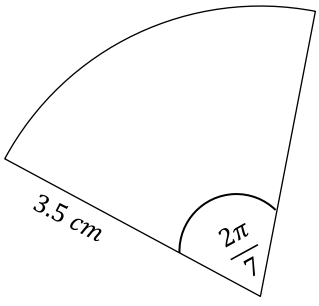


(A) π cm (B) $\frac{49}{4}\pi$ cm (C) $\left(\frac{2}{7}\pi + 7\right)$ cm (D) $(\pi + 7)$ cm

5. The point $P(-2,3)$ lies on $y = f(x)$. What are the coordinates of the image point, P' on the graph of $y = 3f(x-4)+1$

$x' = -2+4$
$= 2$

$y' = 3\times 3+1$
$= 10$

(A) $(2,10)$ (B) $(-6,10)$ (C) $(-6,2)$ (D) $(2,0)$

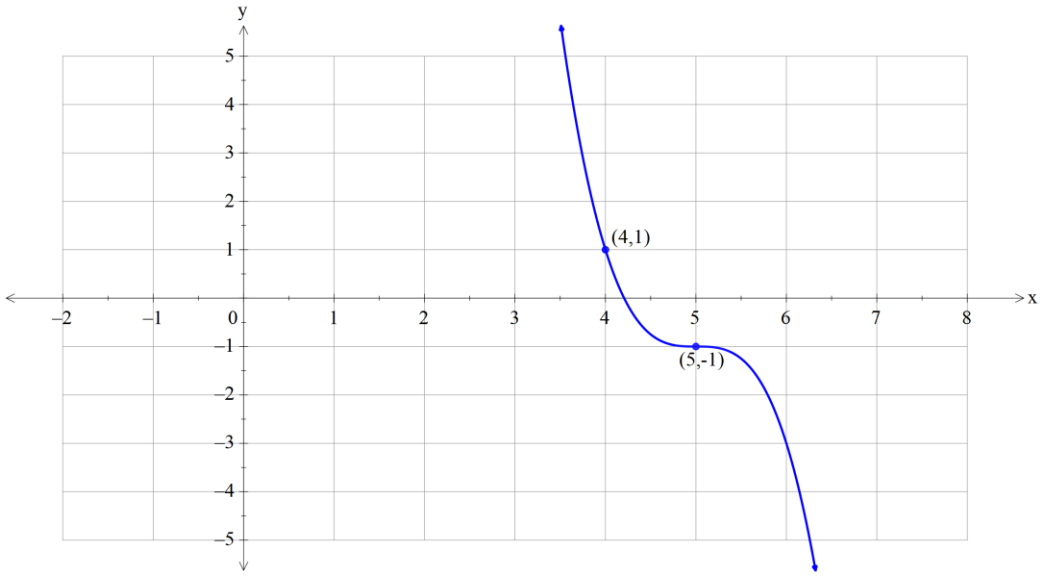
6. Pearl invests \$12000 into a saving account that offers an interest rate of 4.15% p.a. compounded yearly. Gary invests \$12000 into an account with a different interest rate that compounds monthly. What interest rate as a % $p.a.$ does Gary's bank offer if after 4 years both Pearl and Gary have the same amount in their accounts?

$12000(1.0415)^4 = 14119.47$

(A) 4.42% $p.a.$ (B) 4.07% $p.a.$ (C) 3.98% $p.a.$ (D) 3.04% $p.a.$

$\therefore 14119.47 = 12000\left(1+\frac{r}{12}\right)^{48}$

7. The cubic function graphed below has an equation in the form $f(x) = a(x-b)^3 + c$.



a is negative

$c = -1$

$b = 5$

$1 = a(4-5)^3 - 1$

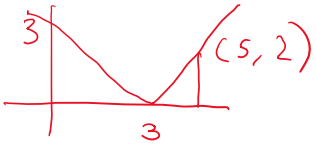
$a = -2$

Which of the following statements is true about $f(x)$.

(A) $a < b < c$ (B) $a < c < b$

(C) $c < a < b$ (D) $b < c < a$

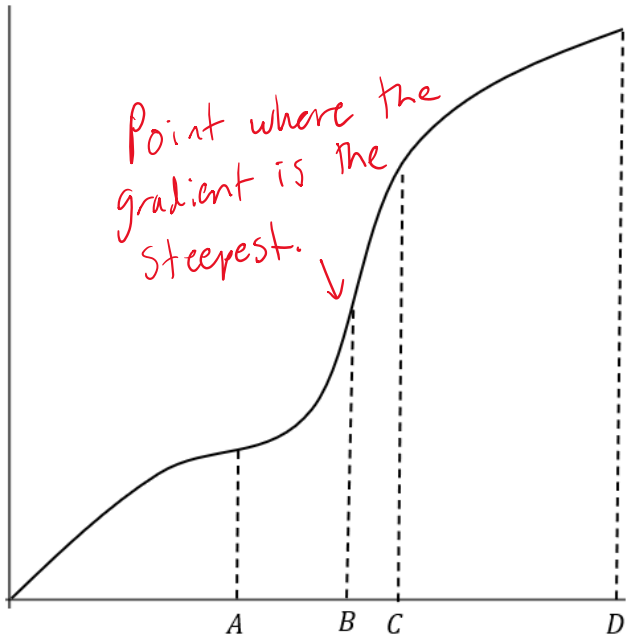
8. Find the value of $\int_0^5 |x-3|\,dx$.



$\frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 2 \times 2$
$= 6.5$

(A) -2.5 (B) 0 (C) 6.5 (D) 13

9. The cumulative distribution function for the continuous random variable X is graphed below. What is the mode of the probability density function?

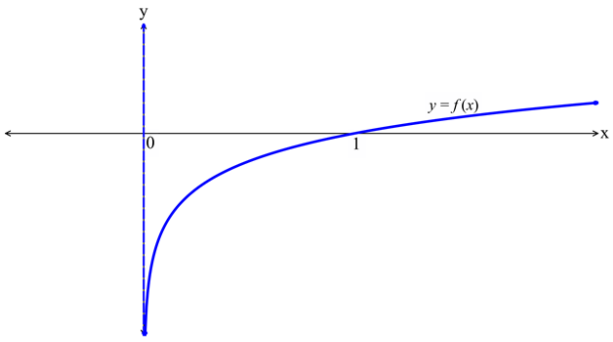


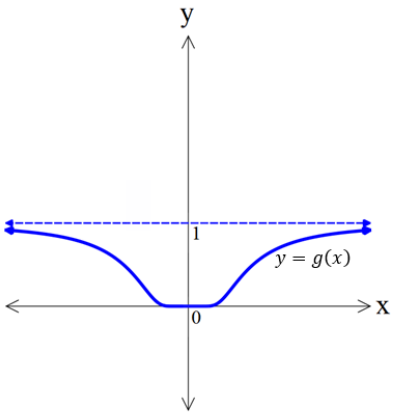
Point where the gradient is the steepest.

So that when differentiated, it becomes the max

(A) A (B) B (C) C (D) D

10. The graphs of $f(x)$ and $g(x)$ are shown below.





range of $g(x)$: $[0, 1)$

What is the range of $f(g(x))$?

range of $f(x)$ in the domain $[0, 1)$:

(A) Range: $(-\infty, 0)$ $(-\infty, 0)$

(B) Range: $(0, \infty)$

(C) Range: $(-\infty, 1)$

(D) Range: $(1, \infty)$

End of Multiple Choice.

Mathematics Advanced

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- Answer each question in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Additional writing space is available on pages 27 to 33. If you use this space. Please start a new sheet of paper for each question.

Question 11 (2 marks)

Solve $\frac{3x}{x+2}=4$. **1**

$3x = 4(x+8)$

$x = -8$ ✓

Question 12 (4 marks)

Evaluate the following expressions.

(a) $\log_2 4x+\log_2 8x-2\log_2 x$ **2**

$= \log_2(32x^2) - \log_2 x^2$ ✓

$= \log_2\left(\frac{32x^2}{x^2}\right)$

$= \log_2 32 = 5$ ✓

(b) $\frac{25^x \times 3^{2x+5}}{15^{2x+1}}$ **2**

$\frac{5^{2x} \times 3^{2x+5}}{5^{2x+1} \times 3^{2x+1}}$ ✓

$= 5^{2x-(2x+1)} \times 3^{2x+5-(2x+1)}$

$= 5^{-1} \times 3^4$ ✓ $= \frac{81}{5}$

Question 13 (2 marks)

Evaluate $\int \frac{x}{(2-x^2)^3}\,dx$. **2**

$\int x(2-x^2)^{-3}\,dx$

$= -\frac{1}{2}\int -2x(2-x^2)^{-3}$ ✓

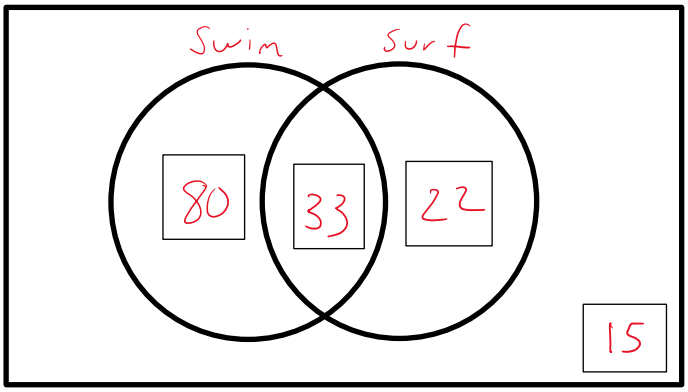
$= +\frac{1}{4}(2-x^2)^{-2} + C$ ✓

$-\frac{1}{2}$ ✓ for $+C$ on this Q

Question 14 (3 marks)

A survey of 150 people found that 113 people liked swimming, 55 liked surfing and 15 liked neither.

(a) Use this information to fill in the blank spaces in the Venn diagram. **2**



$\frac{1}{2}$ mark per box

(b) If 2 people are selected at random, find the probability that both people like swimming only. **1**

$\frac{80}{150} \times \frac{79}{149} = \frac{632}{2235}$ ✓

Question 15 (3 marks)

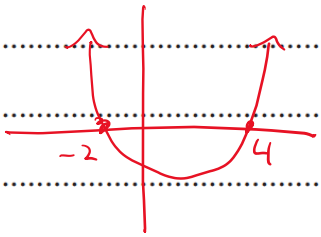
For what values of x is the function $f(x) = x^4 - 4x^3 - 48x^2$ concave down? **3**

$f'(x) = 4x^3 - 12x^2 - 96x$

$f''(x) = 12x^2 - 24x - 96 < 0$ ✓

$x^2 - 2x - 8 < 0$

$(x-4)(x+2) < 0$ ✓

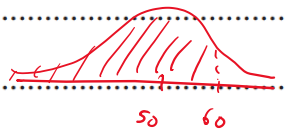


$-2 < x < 4$ ✓

Question 16 (4 marks)

(a) Chloe sits a marine biology test and receives a mark of 60. The mean mark on the test was 50 with a standard deviation of 10. What percentage of her cohort did she score higher than? **2**

Z score: $\frac{60-50}{10} = 1$



✓

$50 + 34 = 84\%$ ✓

(b) Chloe also sits a test on mythological creatures. The mean mark is 57 with a standard deviation of 13. Chloe received a mark of 71. Which test did she do better in compared with her cohort? Justify your answer. **2**

Z Score $_{mc}$: $\frac{71-57}{13} = 1.07$

She did better in mythological creatures because she got a higher z score ✓

Question 17 (6 marks)

A table of future value interest factors for an annuity of $1 is shown.

Table of future value interest factors

n/r	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%
4	4.06040	4.09090	4.12161	4.15252	4.18363	4.21494	4.24646	4.27819
8	8.28567	8.43284	8.58297	8.73612	8.89234	9.05169	9.21423	9.38001
12	12.68250	13.04121	13.41209	13.79555	14.19203	14.60196	15.02581	15.46403
16	17.25786	17.93237	18.63929	19.38022	20.15688	20.97103	21.82453	22.71934
20	22.01900	23.12367	24.29737	25.54466	26.87037	28.27968	29.77808	31.37142
24	26.97346	28.63352	30.42186	32.34904	34.42647	36.66653	39.08260	41.68920
28	32.12910	34.48148	37.05121	39.85980	42.93092	46.29063	49.96758	53.99333
32	37.49407	40.68829	44.22703	48.15028	52.50276	57.33450	62.70147	68.66625

(a) Ariel wants to have saved $10 000 by the end of 8 years. She will make regular instalments at the end of each 6-month period into a bank account with 4% interest p.a. compounded every 6 months. Find the value of each instalment that Ariel needs to make to reach her goal. **2**

$10000 = 18.63929 \times M$

$M = \$536.50$ ✓

(b) Show that the amount in the account after 2 years is $2211.24. **1**

536.50×4.12161

$= \$2211.24$ ✓

(c) After 2 years, Ariel gets a promotion at work. She can now afford to make the same size instalments as in part (a) on a quarterly basis. The interest is still 4% p.a. but compounded quarterly. How much more than her goal of $10 000 will she have at the end of the 8 years? **3**

$2211.24 \times (1.01)^{6\times4} + 536.5 \times 26.97346$ ✓ ✓

$= \$17278.95$ ✓

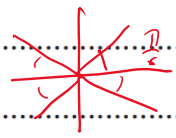
$\therefore$ exceeds her goal by $\$7278.95$

Question 18 (3 marks)

Solve the equation $(2\cos x)^2 = 3$ in the domain $-\pi \leq x \leq \pi$. **3**

$2\cos x = \pm\sqrt{3}$

$\cos x = \pm\frac{\sqrt{3}}{2}$ ✓

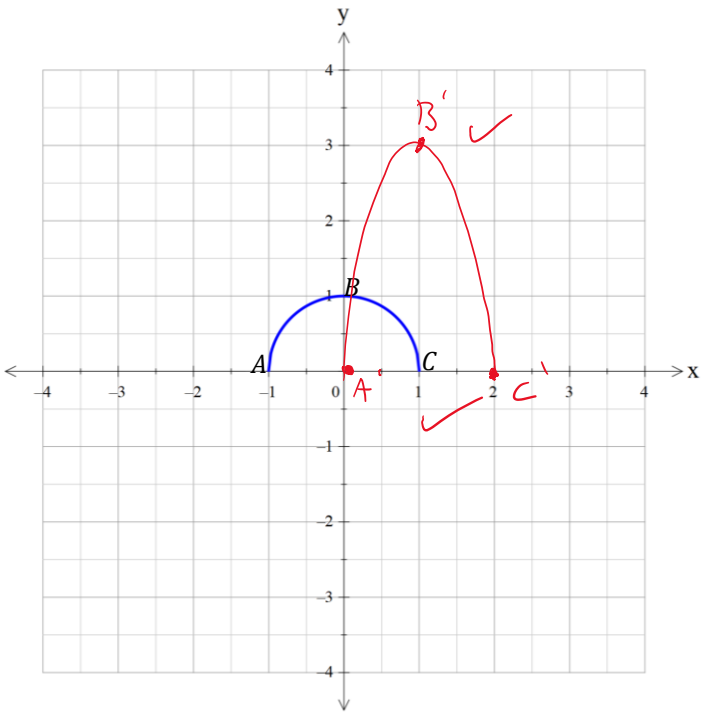


$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{6}, -\frac{5\pi}{6}$ ✓ ✓

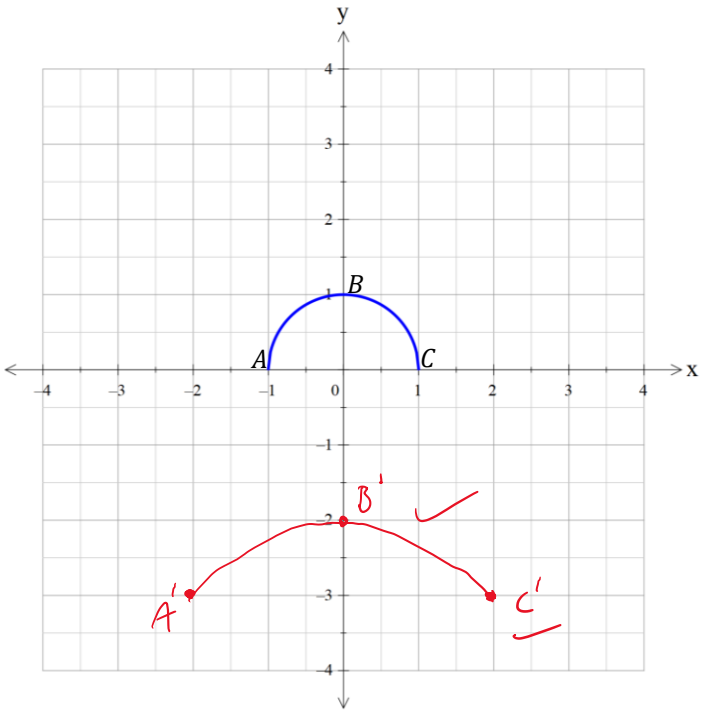
Question 19 (4 marks)

The function $y = f(x)$ is graphed below. On the same set of axes, sketch the transformed function clearly showing the transformed points of A, B and C.

(a) $y = 3f(x-1)$ **2**



(b) $y = f\left(\frac{1}{2}x\right) - 3$ **2**



Question 20 (5 marks)

(a) Find the equation of the normal to the curve $y = -x^2 + 4x - 1$ at the point $(1,2)$. **3**

$y' = -2x + 4$

m at $x = 1$, $y' = -2 + 4$

$= 2$ ✓

Perpendicular $M = -\frac{1}{2}$

equation: $y - 2 = -\frac{1}{2}(x - 1)$ ✓

$y - 2 = -\frac{x}{2} + \frac{1}{2}$

$y = -\frac{x}{2} + \frac{5}{2}$ ✓

(b) The normal intersects $y = -x^2 + 4x - 1$ again at point P. Find the coordinates of point P. **2**

$-\frac{x}{2} + \frac{5}{2} = -x^2 + 4x - 1$

$-x + 5 = -2x^2 + 8x - 2$

$2x^2 - 9x + 7 = 0$

$2x^2 - 2x - 7x + 7 = 0$

$(x - 1)(2x - 7) = 0$ ✓

$x = 1, \quad x = \frac{7}{2}$

$P\left(\frac{7}{2}, \frac{3}{4}\right)$ ✓

Question 21 (3 marks)

Show that $\frac{1-tan^2x}{1+tan^2x}=1-2sin^2x$. **3**

LHS: $\frac{1-\tan^2x}{\sec^2x}$ ✓

$= \frac{1}{\sec^2x} - \frac{\tan^2x}{\sec^2x}$

$= \cos^2x - \frac{\frac{\sin^2x}{\cos^2x}}{\frac{1}{\cos^2x}}$ ✓

$= \cos^2x - \sin^2x$

$= 1 - \sin^2x - \sin^2x$ ✓

$= 1 - 2\sin^2x$

$= RHS$

Question 22 (2 marks)

A geometric sequence is formed by $f(x)+f'(x)+f''(x)+f'''(x)+\cdots$ where $f(x)=e^{\frac{1}{2}x}$. Find the limiting sum in terms of x. **2**

$f'(x) = \frac{1}{2}e^{\frac{1}{2}x}$

$f''(x) = \frac{1}{4}e^{\frac{1}{2}x}$

$\therefore r = \frac{1}{2}$ ✓

$S_\infty = \frac{a}{1-r}$

$= \frac{e^{\frac{1}{2}x}}{1-\frac{1}{2}}$

$= 2e^{\frac{1}{2}x}$ ✓

Question 23 (5 marks)

The population (P) of sea turtles over time (t) in years can be modeled using the equation

$$P = 30000e^{-kt} + 8000.$$

After 10 years, the population has decreased to 26196 turtles.

(a) What is the initial population of sea turtles? **1**

$t=0$

$P = 30000e^{-k\times 0} + 8000$

$= 38000$ ✓

(b) Show that the value of k is 0.05 rounded correct to two decimal places. **2**

$t=10,\ P=26196$

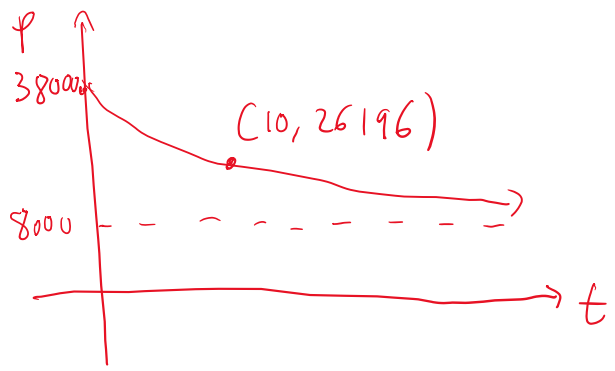
$26196 = 30000e^{-k\times 10} + 8000$ ✓

$18196 = 30000e^{-10k}$

$-10k = \ln\frac{18196}{30000}$ ✓

$k = 0.05$

(c) Sketch the population of sea turtles over time. **2**



½ Shape
½ asymptote
½ extra point
½ y-int

Question 24 (5 marks)

A ball is thrown upwards from the edge of a cliff and hits the ground below. Its vertical height (h) in metres above the ground is given by the formula $h = -t^2 + 2t + 3$ where t is the time in seconds.

(a) At what time does the ball hit the ground? **1**

$0 = -t^2 + 2t + 3$

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0$ ✓

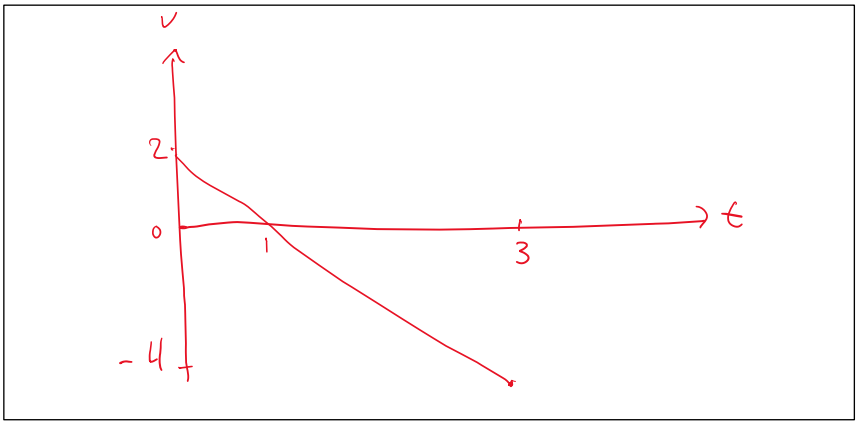
$\therefore t = 3$ or $t = -1$ but $t > 0$ $\therefore t = 3$

(b) At what time is the ball stationary? **1**

$h' = -2t + 2 = 0$

$t = 1$ ✓

(c) In the box below, graph the velocity function of the ball from the time it is launched until it hits the ground. **2**



✓ End points

½ Shape

½ x int

(d) Find the total vertical distance travelled by the ball. **1**

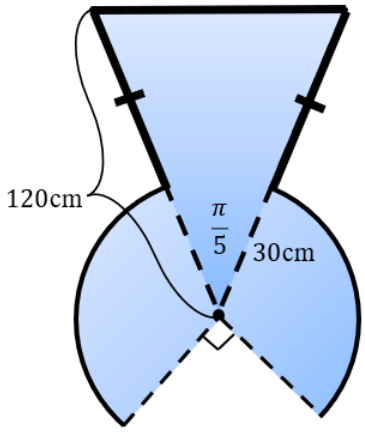
distance $= \frac{1}{2} \times 1 \times 2 + \frac{1}{2} \times 2 \times 4$

$= 5$m ✓

Question 25 (4 marks)

A mermaid tail pictured below is designed using the following:

- A large circle with a 30cm radius.
- An isosceles triangle with 2 sides of length 120 cm and an included angle of $\frac{\pi}{5}$.
- A sector with a 30 cm radius and an included angle of $\frac{\pi}{2}$ that has been removed from the larger circle.
- The vertex of the triangle and the sector meet at the centre of the circle.



NOT TO SCALE

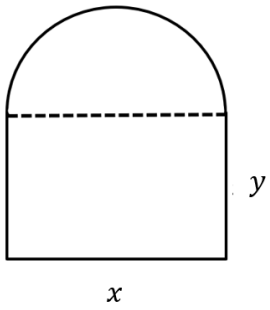
Find the area of the mermaid tail (the shaded area), giving your answer correct to 1 decimal place. **4**

A = Circle + triangle − sector − sector

$= 30^2\pi + \frac{1}{2}\times 120^2 \sin\left(\frac{\pi}{5}\right) - \frac{1}{2}\times 30^2 \times \frac{\pi}{5} - \frac{1}{2}\times 30^2 \times \frac{\pi}{2}$

$= 6069.9\ cm^2$

Question 26 (6 marks)
The cross section of a treasure chest is in the shape of a rectangle and a semicircle. The total perimeter of the cross section is 250 cm.



(a) Show that the area of the of the cross section is given by: **3**

$$A = 125x - x^2\left(\frac{\pi+4}{8}\right).$$

$P = 250 = 2y + x + \frac{1}{2}x\pi$ ✓

$2y = 250 - \frac{2x+\pi x}{2}$

$y = 125 - \frac{2x+\pi x}{4}$

$A = xy + \frac{1}{2}\left(\frac{x}{2}\right)^2\pi$ ✓

$A = x\left(125 - \frac{2x+\pi x}{4}\right) + \frac{x^2\pi}{8}$

$= 125x - \frac{4x^2 - 2\pi x^2 + \pi x^2}{8}$

$= 125x - x^2\left(\frac{\pi+4}{8}\right)$ ✓

(b) Find the maximum area of the cross-section of the treasure chest giving your answer correct to one decimal place. **3**

$A' = 125 - 2\left(\frac{\pi+4}{8}\right)x = 0$ ✓

$x = \frac{125}{2\left(\frac{\pi+4}{8}\right)} \approx 70.012\ldots$ ½

$y'' = -2\left(\frac{\pi+\pi}{8}\right) < 0$

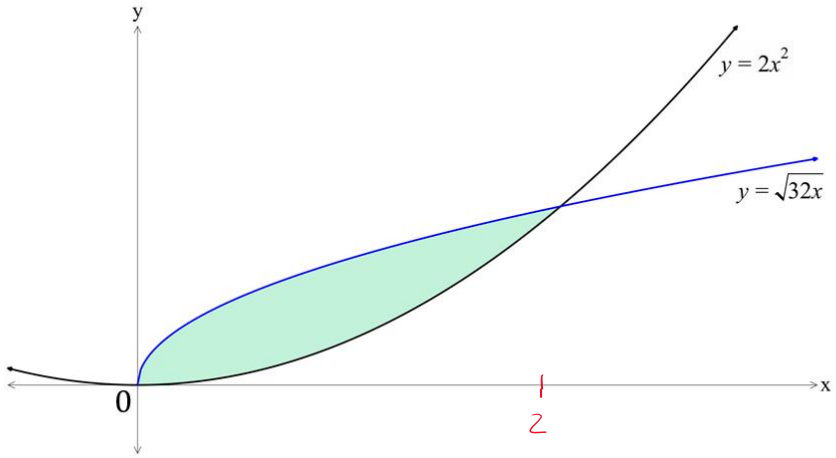
$\therefore$ max ½

$A = 125(70.0) - (70.0)^2\left(\frac{\pi+4}{8}\right)$

$= 4375.8\ cm^2$ ✓

Question 27 (3 marks)

Find the area between the graphs of $y = 2x^2$ and $y = \sqrt{32x}$. **3**



Point of intersection

$2x^2 = \sqrt{32x}$

$4x^4 = 32x$

$x^4 - 8x = 0$

$x = 0, \quad x = 2$ ✓

$\int_0^2 \sqrt{32x} - 2x^2$ ✓½

$\left[\frac{2\sqrt{32}}{3}x^{\frac{3}{2}} - \frac{2}{3}x^3\right]_0^2$ ✓

$= \frac{2\sqrt{32}}{3} \times \sqrt{2^3} - \frac{2 \times 2^3}{3}$

$= \frac{16}{3}$ units^2 ✓½

Question 28 (5 marks)

Eugene deposits a sum of money into a bank account at the start of each year. The bank offers interest of 4% p.a. compounded every 6 months. Let A_n be the value of the account after n years and P be the value of each deposit.

(a) Show that the total value in the account after 3 years is given by: **2**

$$A_3 = P(1.02)^6 + P(1.02)^4 + P(1.02)^2.$$

$A_1 = P(1.02)^2$

$A_2 = (P(1.02)^2 + P)(1.02)^2$

$= P(1.02)^4 + P(1.02)^2$

$A_3 = (P(1.02)^4 + P(1.02)^2 + P)(1.02)^2$

$= P(1.02)^6 + P(1.02)^4 + P(1.02)^2$

(b) Show that $A_n = \frac{2601}{101}P(1.02^{2n} - 1)$. **2**

$A_n = P(1.02)^{2n} + P(1.02)^{2n-2} + P(1.02)^{2n-4} + \ldots + P(1.02)^2$

$= P(1.02)^2\left(1.02^{2n-2} + 1.02^{2n-4} + 1.02^{2n-6} + \ldots + 1\right)$

$= P(1.02)^2 \times \frac{1\left((1.02^2)^n - 1\right)}{1.02^2 - 1}$ $\quad a = 1,\ r = 1.02^2,\ n = n$

$= P(1.02)^2 \times \frac{1.02^{2n} - 1}{1.02^2 - 1}$

$= P\left((1.02)^{2n} - 1\right) \times \frac{2601}{101}$

$= \frac{2601}{101}P(1.02^{2n} - 1)$

Question 28 continues on the next page

(c) If Eugene saves \$500 000 dollars at the end of 25 years, find the size of his deposits. **1**

$n = 25 \qquad A_n = 500\,000$

$500000 = \frac{2601}{101} p((1.02)^{2\times 25} - 1)$

$p = \$11477.74$ ✓

Question 29 (4 marks)

(a) Differentiate $(x-1)(\sin x)$. **2**

$u = x-1 \qquad v = \sin x$ ✓

$u' = 1 \qquad v' = \cos x$

$\frac{d}{dx} = \sin x + (x-1)\cos x$

$= \sin x + x\cos x - \cos x$ ✓ (not required expanded but they need to for part b)

(b) Hence find $\int x\cos x \; dx$. **2**

$\int \sin x\,dx + \int x\cos x\,dx - \int \cos x\,dx = (x-1)(\sin x) + C$ ✓

$-\cos x + \int x\cos x\,dx - \sin x + C_2 = (x-1)(\sin x) + C$

$\int x\cos x = (x-1)(\sin x) + \cos x + \sin x + C_3$ ✓

$= x\sin x - \sin x + \cos x + \sin x + C_3$

$= x\sin x + \cos x + C_3$

No penalty for forgetting $+C$, has been penalised earlier

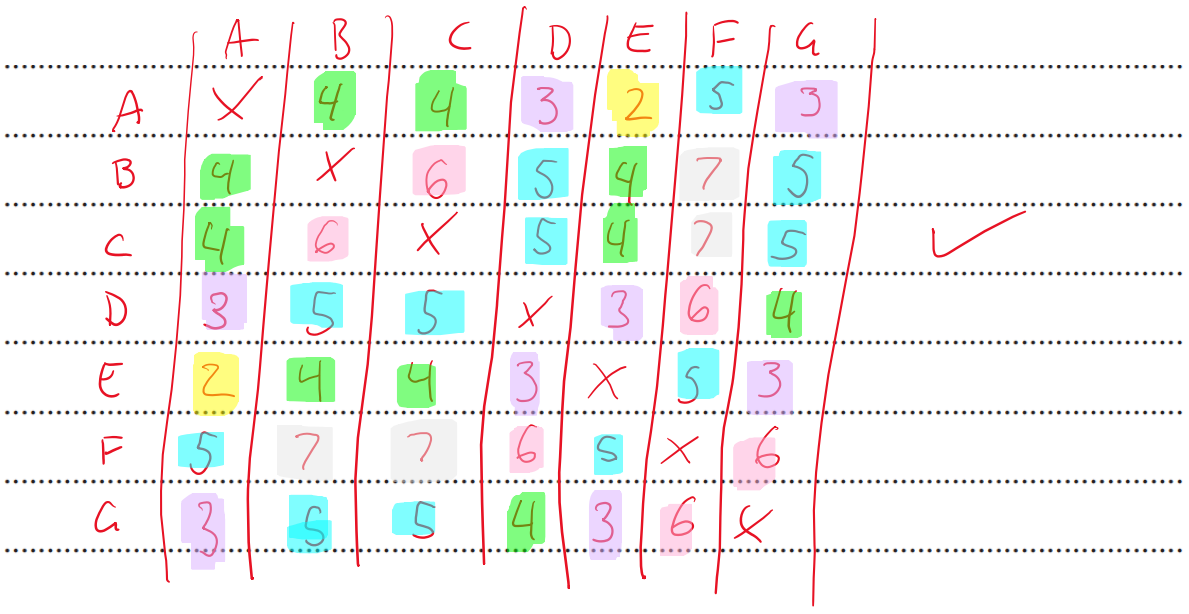
Question 30 (6 marks)

Seven scrabble tiles labelled with the letters A to G are placed in a bag. Each letter has a value assigned to it as shown in the photo below.

A player picks 2 tiles from the bag (1 at a time without replacement), and their score (X) is the sum of the numbers on the tiles.

(a) Complete the discrete probability distribution table by calculating the probability of getting each score. It may help to draw a sample space. **2**

Score	2	3	4	5	6	7
$p(X = x)$	2/42	8/42	10/42	12/42	6/42	4/42



(b) What is the expected score from playing this game? **2**

$E(X) = 2 \times \frac{2}{42} + 3 \times \frac{8}{42} + 4 \times \frac{10}{42} + 5 \times \frac{12}{42} + 6 \times \frac{6}{42} + 7 \times \frac{4}{42} = \frac{32}{7}$

Questions 30 continues on the next page

(c) Sebastian plays this game and scores a 5. What is the probability that he picked a tile with a vowel? **2**

$\frac{4}{12} = \frac{1}{3}$

Question 31 (8 marks)

The depth (d) in metres of the water in a wave pool varies as waves are sent out. The depth is measured from the floor and can be modelled by the equation $d = \frac{4}{5}\sin at + \frac{3}{2}$, where t is the time in seconds and a is a constant. Each wave has a period of 2.5 seconds measured from the peak of the wave to the peak of the next wave.

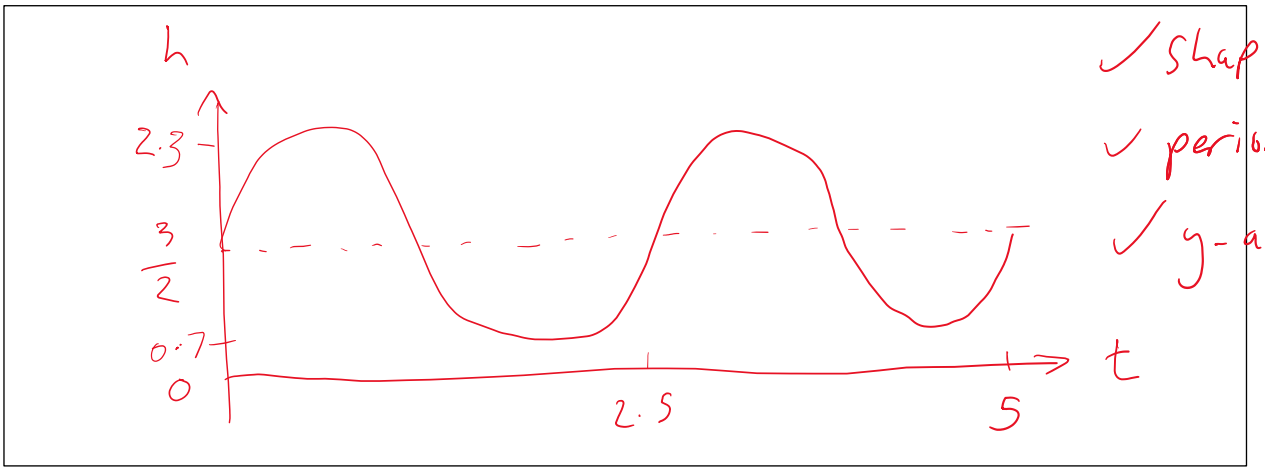
(a) Find the exact value of a. **1**

$\frac{2\pi}{a} = 2.5$, $a = \frac{4\pi}{5}$

(b) Find the maximum height of the wave. **1**

$\frac{4}{5} + \frac{3}{2} = 2.3\text{m}$

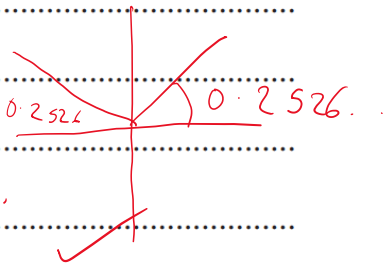
(c) Sketch a graph showing the depth of the water over the first 5 seconds. **3**



(d) Sandy can only touch the bottom of the pool when the height of the wave is less than 1.7m. For what percentage of the first 5 seconds can she touch the bottom of the pool? Give your answer correct to 1 decimal place. **3**

$1.7 = 0.8 \sin \frac{4\pi}{5} t + 1.5$

$\frac{1}{4} = \sin \frac{4\pi t}{5}$ ✓



$\frac{4\pi t}{5} = 0.2526..., \pi - 0.2526...$

$t = 0.1005, 1.149$

$\text{percentage} = \frac{1.149 - 0.1005}{2.5}$ ½

$= 42.0\%$ ½

Can touch $100 - 42.0\% = 58.0\%$ of the time

Question 32 (4 marks)

The lengths of fish in metres in the harbour are distributed according to the equation,

$$f(x) = \begin{cases} -2x(x^2-1)^3 + 0.5 & for \quad 0 < x \leq 1 \\ -\dfrac{x-2}{2} & for\ 1 < x < 2 \\ 0 & for\ x > 2 \end{cases}$$

People are only permitted to catch a fish whose length is between 0.45 m and L m long. If 60% of fish in the harbour are within this legal limit, find the value of L correct to 2 decimal places. **4**

$$\int_{0.45}^{1} -2x(x^2-1)^3 + 0.5\,dx + \int_{1}^{L} -\frac{x-2}{2}\,dx = 0.6 \quad \checkmark$$

$$\left[-\frac{(x^2-1)^4}{4} + 0.5x\right]_{0.45}^{1} + \left[-\frac{x^2}{4} + \frac{2x}{2}\right]_{1}^{L} \quad \checkmark$$

$$0.5 - 0.12387 + \frac{-L^2}{4} + \frac{2L}{2} - \frac{3}{4} = 0.6$$

$$\frac{-L^2}{4} + \frac{2L}{2} - 0.97387 = 0$$

$$L^2 - 4L + 3.89548 = 0$$

$$L = \frac{4 \pm \sqrt{4^2 - 4 \times 3.89548}}{2} \quad \checkmark$$

$$L = 2.32 \quad \text{or} \quad 1.67\text{m}$$

but only 1.67 m is in the domain

∴ fish can be caught to a length of 1.67 m ✓

End of Examination

Additional writing space.
Label each question carefully.

Additional writing space.
Label each question carefully.

Additional writing space.
Label each question carefully.

Additional writing space.
Label each question carefully.

Additional writing space.
Label each question carefully.

Additional writing space.
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